

Deformed photon-added nonlinear coherent states and their nonclassical properties

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Abstract

In this paper, we will try to present a general formalism for the construction of *deformed photon-added nonlinear coherent states* (DPANCSs) $|\alpha, f, m\rangle$, which in special case lead to the well-known photon-added coherent state (PACS) $|\alpha, m\rangle$. Some algebraic structures of the introduced DPANCSs are studied and particularly the resolution of the identity, as the most important property of generalized coherent states, is investigated. Meanwhile, it will be demonstrated that, the introduced states can also be classified in the f -deformed coherent states, with a special nonlinearity function. Next, we will show that, these states can be produced through a simple theoretical scheme. A discussion on the DPANCSs with negative values of m , i.e., $|\alpha, f, -m\rangle$, is then presented. Our approach, has the potentiality to be used for the construction of a variety of new classes of DPANCSs, corresponding to any nonlinear oscillator with known nonlinearity function, as well as arbitrary solvable quantum system with known discrete, nondegenerate spectrum. Finally, after applying the formalism to a particular physical system known as Pöschl-Teller (P-T) potential and the nonlinear coherent states corresponding to a specific nonlinearity function $f(n) = \sqrt{n}$, some of the nonclassical properties such as Mandel parameter, second order correlation function, in addition to first and second-order squeezing of the corresponding states will be investigated, numerically.

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1 Introduction

As was shown by Agarwal and Tara, "photon-added coherent states" (PACSs) are obtained by iterated actions (m times) of bosonic creation operator a^\dagger on the standard coherent states $|\alpha\rangle$. The explicit form of these states has been expressed as follows [1]

$$|\alpha, m\rangle = \frac{\exp(\frac{-|\alpha|^2}{2})}{[L_m(-|\alpha|^2)m!]^{1/2}} \sum_{n=0}^{\infty} \frac{\alpha^n \sqrt{(n+m)!}}{n!} |n+m\rangle, \quad (1)$$

where m is a non-negative integer and $L_m(x)$ is the m th-order of Laguerre polynomial. These states exhibit nonclassical features like squeezing and sub-Poissonian statistics. Besides, the nonlinear coherent state is defined as the solution of the eigenvalue equation

$$A|\alpha, f\rangle = \alpha|\alpha, f\rangle, \quad (2)$$

with the decomposition in the number states space as [2]

$$|\alpha, f\rangle = N(|\alpha|^2)^{-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!} [f(n)]!} |n\rangle, \quad (3)$$

where $n = a^\dagger a$ is the number operator, $A = af(n)$ is the f -deformed annihilation operator, $f(n)$ is an intensity dependent function, $[f(n)]! \doteq f(n)f(n-1)\dots f(1)$ and by convention $[f(0)]! \doteq 1$. It is shown that the states in (1) may be considered as nonlinear coherent states with $f(n, m) = 1 - m/(1+n)$ [3]. Consequently, the eigenvalue equation $f(n, m)a|\alpha, m\rangle = \alpha|\alpha, m\rangle$ is established. Using the Stieltjes power-moment problem, the over-completeness of PACSs is explicitly shown in [4]. Photon-added and photon-subtracted coherent states associated with inverse q -boson operators introduced in [5]. Wave packet dynamics of PACSs has been investigated in [6]. Higher-order squeezing and higher-order sub-Poissonian statistics of PACSs have been studied in [7], and squeezing and higher-order squeezing of PACSs propagating in a Kerr-like medium have been discussed in [8]. Dynamical squeezing of PACSs were investigated in [9]. Generalized hypergeometric photon-added and photon-depleted coherent states introduced in [10] and PACSs for exactly solvable Hamiltonian studied in [11]. Photon-added Barut-Girardello coherent states of the pseudo-harmonic oscillator have been constructed in [12] and recently generation of coherent states of photon-added type via pathway of eigenfunctions has been argued in [14].

On the other side, the experimental scheme for generation of PACSs may be found in the literature. Among them, we may refer to [15] and especially to the recent work of Zavatta *et al* in which $|\alpha, 1\rangle$ has been produced experimentally by using a parametric amplifier consisting of a type I beta-barium borate down-conversion crystal [16].

The goal of the present contribution is to introduce a formalism for the construction of *deformed photon-added nonlinear coherent states* (DPANCSs) by iterated actions of

" f -deformed creation operator" on a "nonlinear coherent state". A deep insight into the works have been down previously in [1-14], in comparison with our work show that, we indeed deformed both a^\dagger (creation operator) and $|\alpha\rangle$ (coherent state), respectively to A^\dagger and $|\alpha, f\rangle$. Hence, our presentation is essentially different with respect to earlier ones, through which we will get new results.

This paper organizes as follows. After the introduction of the explicit form of DPANCSs in the next section, the algebraic structure of the states will be investigated in section 3, from which we will deduce an appropriate nonlinearity function associated with the introduced states. Section 4 deals with the resolution of the identity of the DPANCSs, and then a simple scheme for their generation will be presented in section 5. Then, after applying the proposed approach to Pöschl-Teller potential (P-T) and the nonlinear coherent states corresponding to the nonlinearity function $f(n) = \sqrt{n}$, as some physical realizations of the formalism, the corresponding DPANCSs are introduced and the non-classicality features of the associated states will be numerically investigated, in sections 6. Next, in section 7, DPANCSs with negative values of m are discussed. At last, we conclude the paper in section 8.

2 Introducing the general structure of DPANCSs

Recall that, the actions of f -deformed annihilation and creation operators on the number states expressed, respectively by $A|n\rangle = f(n)\sqrt{n}|n-1\rangle$ and $A^\dagger|n\rangle = f(n+1)\sqrt{n+1}|n+1\rangle$. Following the terminology of Solomon in [17], since we are also working in the deformed quantum optics field and the photons annihilate or create by the actions of the relevant f -deformed ladder operators, the notion of "deformed photon" seems to be reasonable for distinguishing them from usual bosonic counterpart. In this section, we want to introduce a new family of coherent states, has been called by us as DPANCS, using the definition

$$|\alpha, f, m\rangle = N_\alpha^{m,f} A^{\dagger m} |\alpha, f\rangle, \quad m \in \mathbb{Z}^+, \quad (4)$$

where $|\alpha, f\rangle$ is in general any class of nonlinear coherent states introduced in (2) and $N_\alpha^{m,f}$ is an appropriate normalization constant may be determined. It is straightforward to obtain the explicit form of DPANCSs in terms of Fock states by standard procedure. The final result read as

$$\begin{aligned} |\alpha, f, m\rangle &= N_\alpha^{m,f} \left(\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n![f^2(n)]!} \right)^{-\frac{1}{2}} \\ &\times \sum_{n=0}^{\infty} \frac{\alpha^n [f(n+m)]! \sqrt{(n+m)!}}{n![f^2(n)]!} |n+m\rangle, \end{aligned} \quad (5)$$

with the normalization factor

$$N_{\alpha}^{m,f} = \left(\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n![f^2(n)]!} \right)^{\frac{1}{2}} \left(\sum_{n=0}^{\infty} \frac{|\alpha|^{2n} (n+m)! [f^2(n+m)]!}{(n!)^2 [f^4(n)]!} \right)^{-\frac{1}{2}}. \quad (6)$$

In obtaining (5), we have utilized the relation

$$A^{\dagger m} = \frac{[f(n)]!}{[f(n-m)]!} a^{\dagger m}. \quad (7)$$

As a clear fact, notice that, the DPANCSs in (5) reduce to PACSs in (1), when $f(n) = 1$. It is worth mentioning that the number states $\{|0\rangle, |1\rangle, \dots, |m-1\rangle\}$ are absent from the DPANCSs in (5). This situation is exactly similar to PACSs of Agarwal and Tara [1].

3 The algebra structure of DPANCSs

Now, we want to show that, DPANCSs can also be interpreted as f -deformed coherent states with a specific nonlinearity function. This may be done via demonstrating the fact that, the DPANCS may be re-obtained from the eigenvalue equation

$$f_d(n, f, m) a |\alpha, f, m\rangle = \alpha |\alpha, f, m\rangle. \quad (8)$$

Noticing that the non-canonical commutation relation between the f -deformed ladder operators read as

$$[A, A^{\dagger}] = (n+1)f^2(n+1) - nf^2(n), \quad (9)$$

accordingly, it is convenient to show that

$$[A, A^{\dagger m}] = a^{\dagger m-1} \frac{[f(n+m-1)]!}{[f(n)]!} [(n+m)f^2(n+m) - nf^2(n)]. \quad (10)$$

Next, due to the identity

$$a^{\dagger m} f(n) = f(n-m) a^{\dagger m}, \quad (11)$$

the right-hand side of Eq. (10) can be converted to

$$[A, A^{\dagger m}] = g(n, m) A^{\dagger m-1}, \quad (12)$$

where the relation (7) is used and we have set

$$g(n, m) \equiv (n+1)f^2(n+1) - (n-m+1)f^2(n-m+1).$$

Multiplying both sides of equation (2) from the left by $A^{\dagger m}$ yields

$$A^{\dagger m} A |\alpha, f\rangle = \alpha A^{\dagger m} |\alpha, f\rangle. \quad (13)$$

The commutation relation in (12) helps us to rewrite the latter equation as

$$\left(AA^{\dagger m} - g(n, m) A^{\dagger^{m-1}} \right) |\alpha, f\rangle = \alpha A^{\dagger m} |\alpha, f\rangle. \quad (14)$$

At last, making use of the identity $A^{\dagger^{-1}} = \frac{1}{(n+1)f(n+1)}a$ [18] leads us to the following eigenvalue equation

$$\left(f(n+1) - \frac{g(n, m)}{(n+1)f(n+1)} \right) a |\alpha, f, m\rangle = \alpha |\alpha, f, m\rangle. \quad (15)$$

Comparing Eqs. (15) and (8), gives the form of the nonlinearity function associated to DPANCS as follows

$$f_d(n, f, m) = \frac{(n-m+1)f^2(n-m+1)}{(n+1)f(n+1)}, \quad (16)$$

where the nonlinearity function $f(\cdot)$ appeared in the right hand side of (16) is determined by the nonlinearity of the original nonlinear coherent states, $|\alpha, f, m\rangle$ in (5). So, we have finally succeeded in establishing the DPANCSs as f_d -deformed coherent states, too. Clearly, setting $f(n) = 1$ in (16) one readily obtains $f_d(n, f, m) = 1 - \frac{m}{(n+1)}$, which is the nonlinearity function of PACSs [3].

4 Resolution of the identity of DPANCSs

We noticed that, DPANCS in (5) is a superposition of all number states starting with $|m\rangle$. Following the path of [4, 12], the unity operator in such a *subspace* of the total Hilbert space, spanned by the basis $\{|n\rangle\}_{n=m}^{\infty}$ has been written as

$$\hat{I}^{(m)} = \sum_{n=m}^{\infty} |n\rangle \langle n| = \sum_{n=0}^{\infty} |n+m\rangle \langle n+m|. \quad (17)$$

To be precise, the name unity operator for $\hat{I}^{(m)}$ seems to be unsuitable and it is more reasonable to be called the projection operator on the relevant subspace. This operator is bounded and positive valued with a densely defined inverse [13].

So, in such a case which we deal with, the (generalized) resolution of the identity takes the form

$$\frac{1}{\pi} \iint_D d^2\alpha W(|\alpha|^2) |\alpha, f, m\rangle \langle \alpha, f, m| = \hat{I}^{(m)}, \quad (18)$$

where $W(|\alpha|^2)$ is a positive weight function and D expresses the domain of the coherent states centered at the origin of complex plane, both of which may be appropriately determined. Generally, D may be entire plane or a finite disk centered at the origin, depending on the particular chosen $f(n)$. However, since in the continuation of the paper we will

deal with the first type, in what follows we have set infinity in the upper bounds of the integrals, i.e., the Stieltjes moment problem has been encountered. Here, $\alpha = re^{i\varphi}$ and $d^2\alpha \doteq r dr d\varphi$. By substituting equation (5) into (18), one obtains

$$2 \sum_{n=0}^{\infty} \int_0^{\infty} dr r^{2n+1} W(r^2) (N_{\alpha}^{m,f})^2 \left(\sum_{n=0}^{\infty} \frac{r^{2n}}{n! [f^2(n)]!} \right)^{-1} \times \frac{(n+m)! [f^2(n+m)]!}{(n!)^2 [f^4(n)]!} |n+m\rangle \langle n+m| = \hat{I}^{(m)}, \quad (19)$$

where we have utilized $\int_0^{2\pi} d\varphi e^{i\varphi(n-n')} = 2\pi \delta_{nn'}$. Considering the following expression for weight function:

$$W(r^2) = (N_{\alpha}^{m,f})^{-2} \left(\sum_{n=0}^{\infty} \frac{r^{2n}}{n! [f^2(n)]!} \right) r^{2m} \tilde{W}(r^2), \quad (20)$$

we may rewrite (19) as

$$2 \sum_{n=0}^{\infty} \int_0^{\infty} dr r^{2n+2m+1} \tilde{W}(r^2) \frac{(n+m)! [f^2(n+m)]!}{(n!)^2 [f^4(n)]!} \times |n+m\rangle \langle n+m| = \hat{I}^{(m)}. \quad (21)$$

Obviously, to satisfy this equation, the following moment integral should hold

$$2 \int_0^{\infty} dr r^{2n+2m+1} \tilde{W}(r^2) = \frac{(n!)^2 [f^4(n)]!}{(n+m)! [f^2(n+m)]!}. \quad (22)$$

Finally, after performing the variable change $r^2 = x$ and replacing $n+m$ by $k-1$, we arrive at

$$\int_0^{\infty} x^{k-1} \tilde{W}(x) dx = \frac{[(k-m-1)!]^2 [f^4(k-m-1)]!}{(k-1)! [f^2(k-1)]!}. \quad (23)$$

As is clear, prior to investigating this property the explicit form of nonlinearity function, i.e., the particular physical system must be specified.

5 Generation of the DPANCSs

In order to produce the DPANCSs in (5) physically, we consider the slab of excited two-level atoms through a cavity. Let, the initial state of the atom-field system is expressed by $|\Psi(0)\rangle = |\alpha, f\rangle |e\rangle$, where $|e\rangle$ is the excited state of the atom and $|\alpha, f\rangle$ is the nonlinear coherent state field. The interaction Hamiltonian assumes to have the following configuration

$$\mathcal{H} = \hbar g(\sigma_+ A + A^\dagger \sigma_-) \quad (24)$$

where A, A^\dagger are the f -deformed ladder operators and σ_+, σ_- are respectively the raising and lowering operators of atomic states. In other words, a deeper insight into our proposed Hamiltonian in (24) shows that we have changed the coupling constant g to an alternative coupling $gf(n)$, i.e., our setup works with an intensity dependent coupling. The initial state $|\Psi(0)\rangle$ evolves in time according to

$$|\Psi(t)\rangle = \exp[-i\eta(\sigma_+A + A^\dagger\sigma_-)] |\Psi(0)\rangle, \quad (25)$$

where we have set $\eta \equiv gt$ and g is the coupling constant. For $\eta \ll 1$ one has

$$|\Psi(t)\rangle \simeq (1 - i\eta(\sigma_+A + A^\dagger\sigma_-)) |\alpha, f\rangle |e\rangle. \quad (26)$$

Thus, we will have the simple form of the state vector of the whole atom-field system as follows

$$|\Psi(t)\rangle = |\alpha, f\rangle |e\rangle - i\eta A^\dagger |\alpha, f\rangle |g\rangle. \quad (27)$$

Therefore, if the atom is detected in the ground state $|g\rangle$, then the state of the field is transferred to $A^\dagger |\alpha, f\rangle$, which is indeed the DPANCS $|\alpha, f, 1\rangle$. Hence, we could, in principle, produce the state $|\alpha, f, 1\rangle$. Generalizing the above procedure, one can easily produce, in principle, DPANCSs with arbitrary values of m , by using the Hamiltonian

$$\mathcal{H}_m = \hbar g(\sigma_+ A^m + A^{\dagger m} \sigma_-). \quad (28)$$

Clearly, the state $|\alpha, f, m\rangle$ can be produced using an appropriate m -photon medium.

6 Physical properties of the DPANCSs

In this section we briefly explain some of the ordinarily helpful criteria in the relevant literature, which will be used for investigating the non-classicality exhibition of our introduced states. Along this purpose, we refer to the sub-Poissonian statistics, antibunching phenomenon, quadrature squeezing and finally amplitude squared squeezing. A common feature of all of the above criteria is that the corresponding Glauber Sudarshan P-function of a non-classical state is not positive definite. But, we would like to imply that finding this function is usually a hard task to do. Altogether, each of the above effects, which will be considered in the paper, is in fact, sufficient for a quantum state to belong to non-classical states.

6.1 Non-classicality criteria

Now, we are ready to introduce some of the non-classicality signs which are widely used in the literature. They will help us to investigate the non-classicality features of the introduced states in (5), corresponding to any chosen physical system.

- Photon-counting statistics of the states is investigated by evaluating Mandel parameter has been defined as [24]

$$Q = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle} - 1. \quad (29)$$

The states for which $Q = 0$, $Q < 0$ and $Q > 0$, respectively corresponds to Poissonian (standard coherent states), sub-Poissonian (non-classical states) and super-Poissonian (classical states) statistics.

- Although there are quantum states in which super-/sub-Poissonian statistical behavior is appeared with bunching/antibunching effect, but this is not absolutely true. To investigate bunching or antibunching effects, second-order correlation function is widely used, which is defined as follows [25]:

$$g^2(0) = \frac{\langle a^{\dagger 2} a^2 \rangle}{\langle a^{\dagger} a \rangle^2}. \quad (30)$$

Depending on the specific nonlinearity function $f(n)$, has been chosen for the construction of coherent states, $g^2(0) > 1$ ($g^2(0) < 1$) indicates to bunching (antibunching) effect. The case $g^2(0) = 1$ corresponds particularly to the canonical coherent states.

- In order to examine the quantum fluctuations of quadratures of the field, we consider the hermitian operators $x = (a + a^{\dagger})/\sqrt{2}$ and $p = (a - a^{\dagger})/i\sqrt{2}$ with commutation relation $[x, p] = i$. With the help of common definitions of variances of position and momentum, the following parameters may be defined: $s_x = \frac{(\Delta x)^2 - 0.5}{0.5}$ and $s_p = \frac{(\Delta p)^2 - 0.5}{0.5}$, respectively for quadrature squeezing in x and p . These squeezing parameters can be re-written as follow:

$$s_x = 2 \langle a^{\dagger} a \rangle + \langle a^2 \rangle + \langle a^{\dagger 2} \rangle - \langle a \rangle^2 - \langle a^{\dagger} \rangle^2 - 2 \langle a \rangle \langle a^{\dagger} \rangle \quad (31)$$

and similarly for p as

$$s_p = 2 \langle a^{\dagger} a \rangle - \langle a^2 \rangle - \langle a^{\dagger 2} \rangle + \langle a \rangle^2 + \langle a^{\dagger} \rangle^2 - 2 \langle a \rangle \langle a^{\dagger} \rangle. \quad (32)$$

A state is squeezed in x or p if it satisfies the inequalities $-1 \leq s_x < 0$ or $-1 \leq s_p < 0$, respectively.

- Amplitude-squared squeezing [26] is defined in terms of hermitian operators $X = (a^2 + a^{\dagger 2})/2$ and $P = (a^2 - a^{\dagger 2})/2i$. Their commutation relation calculated as $[X, P] = i(2n + 1)$. The squeezing condition in X or P are respectively given by

$-1 \leq S_X < 0$ or $-1 \leq S_P < 0$, where:

$$\begin{aligned} S_X &= \frac{(\Delta X)^2 - \langle n + \frac{1}{2} \rangle}{\langle n + \frac{1}{2} \rangle}, \\ S_P &= \frac{(\Delta P)^2 - \langle n + \frac{1}{2} \rangle}{\langle n + \frac{1}{2} \rangle}. \end{aligned} \quad (33)$$

The numerator of above parameters can be re-written as follows:

$$\begin{aligned} (\Delta X)^2 - \langle n + \frac{1}{2} \rangle &= \frac{1}{4} \left(\langle a^{\dagger 4} \rangle + \langle a^4 \rangle + 2 \langle a^{\dagger 2} a^2 \rangle \right) \\ &\quad - \frac{1}{4} \left(\langle a^{\dagger 2} \rangle + \langle a^2 \rangle \right)^2, \end{aligned} \quad (34)$$

$$\begin{aligned} (\Delta P)^2 - \langle n + \frac{1}{2} \rangle &= \frac{1}{4} \left(-\langle a^{\dagger 4} \rangle - \langle a^4 \rangle + 2 \langle a^{\dagger 2} a^2 \rangle \right) \\ &\quad + \frac{1}{4} \left(\langle a^{\dagger 2} \rangle - \langle a^2 \rangle \right)^2. \end{aligned} \quad (35)$$

All of the necessary expectation values for computing Q , $g^2(0)$, s_x , s_p , S_X and S_P , corresponding to DPANCSs with arbitrary nonlinearity function $f(n)$ have been presented in the Appendix A.

6.2 Physical properties of the DPANCSs associated with Pöschl-Teller (P-T) potential

In this subsection, we want to apply the presented mathematical-physics structure of DPANCSs in section (2) to a well-known physical system, i.e., P-T potential, which has its specific importance in atomic and molecular physics (see [19] and references therein). This system possesses the following non-degenerate spectrum $e_n = n(n + \nu)$, $\nu > 2$. The special case $\nu = 2$ characterizes the one dimensional square potential well. The nonlinearity function corresponding to this system according to the proposed formalism in [20, 21] may be easily obtained as

$$f(n) = \sqrt{n + \nu}. \quad (36)$$

Inserting (36) in (5), one can easily create the explicit form of DPANCSs associated to P-T potential. We continue our study by discussing some of the quantum statistical properties and non-classicality features of the DPANCSs associated with the mentioned system. This investigation seems to be necessary, due to the fact that even though the nonlinear coherent states mostly possess less or more of the non-classicality signs, but there exists nonlinear coherent states which do not show neither of the usual non-classicality criteria [23]. Altogether, before paying attention to this subject, we would like to establish the resolution of the identity requirement for the introduced states.

6.2.1 Resolution of the identity for the DPANCSs of P-T potential:

Due to the central importance of the resolution of the identity for any class of coherent states, we examine this property according to (23) by using the nonlinearity function of P-T potential, i.e.,

$$\int_0^\infty x^{k-1} \tilde{W}(x) dx = \frac{(m+\nu)! [\Gamma(k-m)]^2 [\Gamma(k-m+\nu)]^2}{(\nu!)^2 \Gamma(k+\nu) \Gamma(k)}. \quad (37)$$

With the help of definition of Meijer's G -function, it follows that [22]

$$\begin{aligned} \int_0^\infty dx x^{k-1} G_{p,q}^{m,n} \left(\beta x \left| \begin{matrix} a_1, \dots, a_n, a_{n+1}, \dots, a_p \\ b_1, \dots, b_m, b_{m+1}, \dots, b_q \end{matrix} \right. \right) \\ = \frac{1}{\beta^k} \frac{\prod_{j=1}^m \Gamma(b_j + k) \prod_{j=1}^n \Gamma(1 - a_j - k)}{\prod_{j=m+1}^q \Gamma(1 - b_j - k) \prod_{j=n+1}^p \Gamma(a_j + k)}. \end{aligned} \quad (38)$$

Comparing equations (37) and (38), it is easy to obtain

$$\tilde{W}(x) = \frac{(m+\nu)!}{(\nu!)^2} G_{2,4}^{4,0} \left(x \left| \begin{matrix} 0, \nu \\ -m, -m, \nu - m, \nu - m \end{matrix} \right. \right). \quad (39)$$

Therefore, via using the above results in (20) and after setting $|\alpha|^2 = x$, the weight function finally takes the form

$$W(x) = G_{2,4}^{1,2} \left(-x \left| \begin{matrix} -m, -\nu - m \\ 0, 0, -\nu, -\nu \end{matrix} \right. \right) G_{2,4}^{4,0} \left(x \left| \begin{matrix} m, \nu + m \\ 0, 0, \nu, \nu \end{matrix} \right. \right), \quad (40)$$

where we have utilized the relation [12]

$$x^s G_{p,q}^{m,n} \left(x \left| \begin{matrix} (a_p) \\ (b_q) \end{matrix} \right. \right) = G_{p,q}^{m,n} \left(x \left| \begin{matrix} (a_p + s) \\ (b_q + s) \end{matrix} \right. \right) \quad (41)$$

and

$$\begin{aligned} N_\alpha^{m,f} &= [\nu! x^{-\nu/2} I_\nu(2\sqrt{x})]^{-\frac{1}{2}} \\ &\times \left[\frac{(\nu!)^2}{(m+\nu)!} G_{2,4}^{1,2} \left(-x \left| \begin{matrix} -m, -\nu - m \\ 0, 0, -\nu, -\nu \end{matrix} \right. \right) \right]^{-\frac{1}{2}}, \end{aligned} \quad (42)$$

which is indeed the closed form of normalization factor for the DPANCSs corresponding to P-T potential, and $I_\nu(x)$ is the modified Bessel function of the first kind.

6.2.2 Numerical results of the DPANCSs associated with P-T potential:

Using the mentioned criteria in subsection 6.1, we will argue and investigate the non-classicality of DPANCSs associated with P-T potential. This can be considered as a

physical realization of our proposed formalism. Firstly, figure 1 displays weight function versus x for different values of m and fixed value of $\nu = 3$. These results signify the positivity of $W(x)$. It is seen that $W(x)$ has a singularity at $x = 0$ but it tends to zero for $x \rightarrow \infty$. In figure 2, $W(x)$ has been plotted versus x for fixed $m = 1$ and various values of ν . The general behavior of weight function in this case is the same as figure 1. We continue with investigating the non-classicality of the introduced states. For this purpose, by using the required mean values (see Appendix A), we have plotted Mandel parameter, second-order correlation function, quadrature squeezing and amplitude-squared squeezing for the DPANCSs of P-T potential versus real α , for various values of m and fixed $\nu = 3$. As is shown in figure 3, Mandel parameter always is negative and so the sub-Poissonian behavior is visible. It is clearly seen that this parameter for the DPANCSs of P-T potential are more negative than PACSs ($f(n) = 1$). So, our deformation increases the depth of the non-classicality of these states. Besides, increasing m results in increasing the non-classicality of DPANCSs in analogously to the results of the PACSs. Meanwhile, for large values of α in both of PACSs and DPANCSs, the Q parameters coincide with each other for different chosen values of m . It indeed tends to a finite negative value for large α . According to figure 4, it is visible that $g^2(0) < 1$ for small values of α and so antibunching effect occurs. This observation illustrates that sub-Poissonian statistics and antibunching effect occur simultaneously in this finite range, as one may compare figures 3 and 4. But, our further calculations for larger α (and certainly the same fixed parameters) show that while Mandel parameter tends to a finite negative value (≈ -0.5), the correlation function tends to ≈ 1 , corresponds to correlation function of canonical coherent state. So, while the states in hand have sub-Poissonian statistics, they do not show antibunching effect for large α . It is noticeable that, in this case, comparing the two distinct non-classicality criteria, the Mandel parameter is more sensitive than the second-order correlation function. Squeezing parameters have been plotted in figure 5. We conclude from our numerical results which presented in figure 5-a that, the DPANCSs are squeezed in x -quadrature, in a wide region of α , with no squeezing in p -quadrature (see figure 5-b). It is evident that for large values of α , s_x and s_p respectively tends to -0.5 and 1 . From figure 5-c it is visible that in some regions of space S_X gets negative values, i.e., amplitude-squared squeezing in X appears. But, as it is observed from figure 5-d, S_P is always positive, i.e., no amplitude-squared squeezing in P may be seen. Our further calculations confirm that with increasing the values of α , S_X and S_P respectively tend to ≈ -0.5 and ≈ 1 . Obviously, all of the limiting quantities are correct for the mentioned fixed parameters.

6.3 Numerical results of the DPANCSs for a nonlinear coherent state with $f(n) = \sqrt{n}$

As second example, we work with the original nonlinear coherent states corresponding to the nonlinearity function $f(n) = \sqrt{n}$. The physical interest in the nonlinear coherent states constructed by this function comes out from the fact that it, indeed, rises in a natural way in Hamiltonians illustrating the interaction with intensity-dependent coupling between a two-level atom and an electromagnetic radiation field [27, 28]. Considering this nonlinearity function, the weight function may be straightforwardly obtained as follows:

$$W(x) = (m!)^2 {}_2F_3(1+m, 1+m; 1, 1, 1; x) G_{2,4}^{4,0} \left(x \left| \begin{matrix} m, m \\ 0, 0, 0, 0 \end{matrix} \right. \right), \quad (43)$$

where ${}_pF_q(a; b; x)$ is the generalized hypergeometric function. Figure 6 displays the weight function versus x for different values of m . The positivity of $W(x)$ is revealed which confirms that the obtained DPANCS are in fact of coherent states type, in its exact meaning.

Our aim is producing the DPANCSs associated with this particular system and investigate their physical properties. Inserting the function $f(n) = \sqrt{n}$ in (5), one can easily create the explicit form of the associated DPANCSs. To proceed further, one needs to use the relations which are presented in Appendix A, setting $f(n) = \sqrt{n}$. For this purpose, the normalization factor of the related DPANCSs is required, which may be determined as:

$$N_{\alpha}^{m,f} = \sqrt{\frac{I_0(2\sqrt{x})}{m!}} [{}_2F_3(1+m, 1+m; 1, 1, 1; x)]^{-1/2}, \quad (44)$$

where $I_0(x)$ is the modified Bessel function of the first kind and ${}_pF_q(a; b; x)$ is the generalized hypergeometric function. Henceforth, we are now ready to continue with investigating the non-classicality of the associated states. We have plotted Mandel parameter, second-order correlation function, quadrature squeezing and amplitude-squared squeezing for the corresponding DPANCSs versus real α , for various values of m . Mandel parameter, has been shown in figure 7, is always negative and so the sub-Poissonian behavior is visible. It is clearly seen that this parameter for the DPANCSs of the chosen nonlinearity function is more negative than PACSs ($f(n) = 1$). Therefore, our new deformation also increases the depth of the non-classicality. Besides, increasing m results in increasing the non-classicality of DPANCSs in analogously to the numerical results of PACSs and DPANCSs for P-T potential. With increasing α in both of PACSs and DPANCSs, the corresponding Q parameters coincide with each other for different chosen values of m . Interestingly, it is worth noticing that, while in the case of PACSs for large α , Q tends to zero (non-classicality disappears), this is not so for DPANCSs, again showing the strong non-classicality behavior of the introduced states. According to figure 8, it is visible

that $g^2(0) < 1$ for enough small values of α and so antibunching effect will be appeared. This observation illustrates that sub-Poissonian statistics and antibunching effect occur simultaneously in this finite range, as one may compare figures 7 and 8. But, our further calculations for larger α show that while Mandel parameter tends to a finite negative value (≈ -0.6), the correlation function tends to ≈ 1 , corresponds to correlation function of canonical coherent state. So, the presented states have sub-Poissonian statistics with no antibunching effect for large α . Therefore, we may conclude that comparing the above two non-classicality criteria, Mandel parameter is more sensitive than the second-order correlation function. Squeezing parameters have been plotted in figure 9. It is obvious from our numerical results presented in figure 9-a that the corresponding DPANCSs are squeezed in x -quadrature, in a wide region of $\alpha \geq 1.75$, while no squeezing is seen in p -quadrature (see figure 9-b). It is also evident that for large values of α , s_x and s_p respectively tend to -0.5 and 1 , for those chosen values of m . From figure 9-c it is visible that in some regions of space, especially large values of α , S_X gets negative values, i.e., amplitude-squared squeezing in X appears. But, as it is observed from figure 9-d, S_P is always positive, i.e., no amplitude-squared squeezing in P may be seen. Our further calculations show that, at least for the chosen parameters, with increasing the values of α , S_X and S_P respectively tend to ≈ -0.5 and ≈ 1 .

7 A discussion on DPANCSs with negative m

The form of $f_d(n, f, m)$ in (16) suggests that, a nonlinearity function can be constructed also for negative integer values of m . The corresponding coherent states associated with this nonlinearity function, denoted by us as $|\alpha, f, -m\rangle$, will be called as "DPANCSs with negative m ". In order to construct these states, consider the following eigenvalue equation

$$\frac{(n+m+1)f^2(n+m+1)}{(n+1)f(n+1)}a|\alpha, f, -m\rangle = \alpha|\alpha, f, -m\rangle, \quad (45)$$

which is obtained simply, by replacing m with $-m$ in (15) together with (16). Following the usual procedure, i.e., by expanding $|\alpha, f, -m\rangle$ in terms of the number states and finding the expansion coefficients, one straightforwardly arrives at

$$|\alpha, f, -m\rangle = N_{\alpha}^{-m,f} \sum_{n=0}^{\infty} \frac{\alpha^n m! \sqrt{n!} [f(n)]! [f^2(m)]!}{(n+m)! [f^2(n+m)]!} |n\rangle. \quad (46)$$

The constant $N_{\alpha}^{-m,f}$ is determined by normalization condition as

$$N_{\alpha}^{-m,f} = \left(\sum_{n=0}^{\infty} \frac{|\alpha|^{2n} n! (m!)^2 [f^2(n)]! [f^4(m)]!}{[(n+m)!]^2 [f^4(n+m)]!} \right)^{-\frac{1}{2}}. \quad (47)$$

Unlike the DPANCSs in (5), the states $|\alpha, f, -m\rangle$ contain a superposition of all Fock states starting with the vacuum state $|0\rangle$. In the limit $\alpha \rightarrow 0$, the state $|\alpha, f, -m\rangle$ reduces to

the vacuum state, but in the same limit, irrespective of the value of m , DPANCS reduces to the number state $|m\rangle$. Also, in the limit $m \rightarrow 0$, both of the states $|\alpha, f, \pm m\rangle$, recover trivially the original nonlinear coherent state $|\alpha, f\rangle$. It is worth to add the point that the states $|\alpha, -m\rangle$ which previously argued in [3], may be reobtained by setting $f(n) = 1$ in $|\alpha, f, -m\rangle$ in (46).

The procedure which we followed in section (6) for investigating the resolution of the identity and obtaining the appropriate weight function of DPANCSs, can be used for the DPANCSs with negative m in (46). Notice that for these states the well-defined unity operator is as usual

$$\hat{I}^{(-m)} = \hat{I} = \sum_{n=0}^{\infty} |n\rangle \langle n|. \quad (48)$$

So, in such a case which we deal with, the resolution of the identity requirement takes the form

$$\frac{1}{\pi} \iint d^2\alpha W^{(-m)}(|\alpha|^2) |\alpha, f, -m\rangle \langle \alpha, f, -m| = \hat{I}, \quad (49)$$

where $W^{(-m)}(|\alpha|^2)$ denotes the non-negative weight function should be determined. In this way, one straightforwardly gets

$$\int_0^\infty dx x^n \tilde{W}^{(-m)}(x) = \frac{((n+m)!)^2 [f^4(n+m)]!}{n! [f^2(n)]!} \quad (50)$$

where

$$\tilde{W}^{(-m)}(x) = (N_\alpha^{-m,f})^2 x^{-m} (m!)^2 [f^4(m)]! W^{(-m)}(x). \quad (51)$$

Investigating the case for particular physical system, i.e., P-T potential, we finally arrive at

$$\begin{aligned} W^{(-m)}(x) &= \frac{(\nu!)^2}{[(m+\nu)!]^4 (N_\alpha^{-m,f})^2 (m!)^2} \\ &\times G_{2,4}^{4,0} \left(x \left| \begin{array}{c} 0, \nu \\ m, m, \nu+m, \nu+m \end{array} \right. \right), \end{aligned} \quad (52)$$

where $N_\alpha^{-m,f}$ has been introduced in (47). The DPANCSs with negative m may be called "coherent states" (according to the Klauder definition), if the weight function $W^{(-m)}(x)$ will be positive in all space. To check this requirement we have plotted $W^{(-m)}(x)$ in figure 10 versus x for fixed value $\nu = 3$ and different values of m . As is shown, unfortunately $W^{(-m)}(x)$ in some region of space gets negative values. So, our results in figure 10 indicate that the DPANCSs with negative m , associated to P-T potential can not be known as coherent state.

Upon the latter results, we motivated to check the above procedure for the PACSs with negative m , as introduced in [3] (the states which may be reproduced by setting $f(n) = 1$ in (46)). Unfortunately, the same conclusion has been obtained, i.e., $W_{PACS}^{(-m)}(x)$ will

get negative values in some regions of space (see figure 11). We have also examined our conclusion for the DPANCSs with negative m associated with other nonlinearity functions. For this purpose we worked with $f(n) = 1/\sqrt{n}$ (harmonious states [29]), $f(n) = 1/\sqrt{n+2\kappa-1}$ (Barut-Girardello coherent states of $SU(1,1)$ group [30]) and $f(n) = \sqrt{n}$. In all the latter cases we obtained the same result, i.e., the positivity of the weight function and so the overcompleteness relation do not justify. We should mention that we investigated the uniqueness of the solution of the moment integrals by examining the Carleman criterion [31].

We end this section with mentioning another example for the latter result may be found in the literature, where the following state has been introduced by Klauder *et al* [32]:

$$|\alpha\rangle = [{}_2F_2(1, 1; 2, 2; |\alpha|^2)]^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{(n+1)^2 n!}} |n\rangle, \quad (53)$$

with ${}_pF_q(a; b; x)$ as the generalized hypergeometric function. As is illustrated there, the weight function for this set of states has been derived as $W(x) = xe^{-x}(x-1)$, which is not trivially a non-negative function in all space. So, even though this class of states is normalizable and continuous in the label, however it is not known as a coherent state, in its exact meaning.

8 Summary and conclusion

In summary, we presented a general formalism for the construction of DPANCSs with the explicit form in (5), which recovers, in special cases, PACSs (setting $f(n) = 1$ in (5)) and canonical coherent states (setting $f(n) = 1$ in (5), together with $m = 0$). The algebraic structure and the resolution of the identity requirement of the introduced states are also illustrated. As in the case of PACSs, we established that the DPANCSs can be specified with a nonlinearity function denoted by us as $f_d(n, f, m)$. Therefore, the introduced DPANCSs are of the f -deformed type, too. We briefly argued that, their physical generation is possible. Then, after applying the formalism to P-T potential, the non-classical properties of the DPANCSs associated with P-T potential are checked through evaluating Mandel parameter, second-order correlation function, as well as first and second-order squeezing, numerically. Along the physical realization of the formalism, we also briefly applied the same procedures which have been done for the P-T potential to a well-known class of nonlinear coherent states with $f(n) = \sqrt{n}$. Generally, we observed much intensity (in depth and domain) of non-classicality signs for the DPANCSs associated with such system in comparison with PACSs of [1]. Then, a discussion on the "DPANCSs with negative m " is presented. According to our results, it is deduced that these latter states do not satisfy the resolution of the identity, appropriately. Indeed, the function which satisfies the related moment integral uniquely determined, altogether the positivity

of the obtained function did not confirmed. Also, we further investigated the existence and positivity of the weight function for the PACSs with negative values of m have been introduced in [3]. Unfortunately, we have found the same conclusion. So, recalling the minimal requirements of any quantum state to be exactly named "coherent state" [32], we may conclude that the "PACSs and DPANCSs with negative m " are not strictly known as coherent states.

Finally, it is worth mentioning that even though we have used only P-T potential and a particular class of nonlinear coherent state as some physical realizations of our proposed structure, its essential potentiality to be used for any class of nonlinear coherent states with known nonlinearity function, in addition to any solvable quantum system with arbitrary discrete spectrum should be clear. So, in this way, a vast new family of DPANCSs can, in principle, be constructed. Apart from the generalized structure of our proposal, it is noticeable that, it is a rather different formalism with new outputs in resultant coherent states and their non-classicality aspects, in comparison with earlier works [1-12].

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9 Appendix

The mean values of the relevant operators over the state $|\alpha, f, m\rangle$, required for our numerical calculations, may be easily obtained as follows:

$$\begin{aligned} \langle a \rangle &= \alpha \left(\tilde{N}_{\alpha}^{m,f} \right)^2 \sum_{n=0}^{\infty} \frac{|\alpha|^{2n} (n+m)! [f^2(n+m)]!}{(n!)^2 [f^4(n)]!} \\ &\times \frac{(n+m+1)f(n+m+1)}{(n+1)f^2(n+1)}, \end{aligned} \quad (54)$$

$$\begin{aligned} \langle a^2 \rangle &= \alpha^2 \left(\tilde{N}_{\alpha}^{m,f} \right)^2 \sum_{n=0}^{\infty} \frac{|\alpha|^{2n} (n+m)! [f^2(n+m)]!}{(n!)^2 [f^4(n)]!} \\ &\times \frac{(n+m+1)(n+m+2)f(n+m+1)f(n+m+2)}{(n+1)(n+2)f^2(n+1)f^2(n+2)}, \end{aligned} \quad (55)$$

$$\begin{aligned} \langle a^4 \rangle &= \alpha^4 \left(\tilde{N}_{\alpha}^{m,f} \right)^2 \\ &\times \sum_{n=0}^{\infty} \frac{|\alpha|^{2n} (n+m+4)! [f(n+m+4)]! [f(n+m)]!}{n!(n+4)! [f^2(n)]! [f^2(n+4)]!}, \end{aligned} \quad (56)$$

$$\langle a^\dagger a \rangle = \left(\tilde{N}_\alpha^{m,f} \right)^2 \sum_{n=0}^{\infty} \frac{|\alpha|^{2n} (n+m)! [f^2(n+m)]!}{(n!)^2 [f^4(n)]!} (n+m), \quad (57)$$

$$\begin{aligned} \langle a^{\dagger^2} a^2 \rangle &= \left(\tilde{N}_\alpha^{m,f} \right)^2 \\ &\times \sum_{n=0}^{\infty} \frac{|\alpha|^{2n} (n+m)! [f^2(n+m)]! (n+m)(n+m-1)}{(n!)^2 [f^4(n)]!}, \end{aligned} \quad (58)$$

$$\langle (a^\dagger a)^2 \rangle = \left(\tilde{N}_\alpha^{m,f} \right)^2 \sum_{n=0}^{\infty} \frac{|\alpha|^{2n} (n+m)! [f^2(n+m)]!}{(n!)^2 [f^4(n)]!} (n+m)^2, \quad (59)$$

where we have set $\tilde{N}_\alpha^{m,f} = N_\alpha^{m,f} \left(\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n! [f^2(n)]!} \right)^{-\frac{1}{2}}$ and $N_\alpha^{m,f}$ determined in (6). Note that, $\langle a^\dagger \rangle$, $\langle a^{\dagger^2} \rangle$ and $\langle a^{\dagger^4} \rangle$ can be obtained by taking the complex conjugate of $\langle a \rangle$, $\langle a^2 \rangle$ and $\langle a^4 \rangle$, respectively.

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FIGURE CAPTIONS

FIG. 1 The plot of $W(x)$ as a function of x , with fixed parameter $\nu = 3$ and different values of m for DAPNCS associated with P-T potential. Continuous line is for $m = 1$, dotted line is for $m = 5$ and dashed line is for $m = 9$.

FIG. 2 The plot of $W(x)$ as a function of x , with fixed parameter $m = 1$ and different values of ν for DAPNCS associated with P-T potential. Continuous line is for $\nu = 7$, dotted line is for $\nu = 5$ and dashed line is for $\nu = 3$.

FIG. 3 The variation of Q as a function of $\alpha \in \mathbb{R}$, with fixed parameter $\nu = 3$ and different values of m for DPANCS associated with P-T potential. Continuous line is for DPANCSs and $m = 2$, dashed line is for DPANCSs and $m = 5$, dot-dashed line is for PACSs and $m = 2$, and dotted line is for PACSs and $m = 5$.

FIG. 4 The variation of $g^2(0)$ as a function of $\alpha \in \mathbb{R}$, with fixed parameters $\nu = 3$ and different values of m for DPANCS associated with P-T potential. Continuous line is for $m = 1$, dashed line is for $m = 2$ and dotted line is for $m = 3$.

FIG. 5 Squeezing parameters as a function of $\alpha \in \mathbb{R}$, with fixed parameter $\nu = 3$ and different values of m for DPANCS associated with P-T potential. (a): shows the variation of s_x , continuous line is for $m = 1$, dotted line is for $m = 2$ and dashed line is for $m = 3$; (b): the same as (a) except that it is plotted for s_p ; (c): shows the variation of S_x , continuous line is for $m = 1$, dotted line is for $m = 3$ and dashed line is for $m = 5$; (d): the same as (c) except that it is plotted for S_p .

Fig. 6 The plot of $W(x)$ as a function of x for DAPNCS associated with $f(n) = \sqrt{n}$. Continuous line is for $m = 1$, dotted line is for $m = 5$ and dashed line is for $m = 9$.

Fig. 7 The parameter Q as a function of $\alpha \in \mathbb{R}$. Continuous line and dashed line are respectively for $m = 2$ and $m = 5$ for DPANCS associated with $f(n) = \sqrt{n}$. The dot-dashed line and dotted line are respectively for $m = 2$ and $m = 5$ for PACSs ($f(n) = 1$).

Fig. 8 The variation of $g^2(0)$ as a function of $\alpha \in \mathbb{R}$ for DPANCS associated with $f(n) = \sqrt{n}$. Continuous line is for $m = 1$, dashed line is for $m = 2$ and dotted line is for $m = 3$.

Fig. 9 Squeezing parameters as a function of $\alpha \in \mathbb{R}$ for different values of m for DPANCS associated with $f(n) = \sqrt{n}$. (a): shows the variation of s_x , continuous line is for $m = 1$, dotted line is for $m = 2$ and dashed line is for $m = 3$; (b): the same as (a) except that it is plotted for s_p ; (c): shows the variation of S_x , continuous line is for $m = 1$,

dotted line is for $m = 3$ and dashed line is for $m = 5$; (d): the same as (c) except that it is plotted for S_p .

FIG. 10 The plot of $W^{(-m)}(x)$ as a function of x , with fixed m parameters and $\nu = 3$ for DPANCS with negative m associated with P-T potential. Continuous line is for $m = 1$, dashed line is for $m = 2$ and dotted line is for $m = 3$.

FIG. 11 The plot of $W_{PACS}^{(-m)}(x)$ as a function of x , with fixed m parameters for PACS with negative m . Continuous line is for $m = 1$, dotted line is for $m = 2$ and dashed line is for $m = 3$.